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Final ONR report on grant N00014-97-1-0095 Stratified Flow, Wave Packet Reflection and Topographic Currents

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A wide range of investigations was carried out under this grant. The topics covered by these investigations include vortex stability, vortex dynamics, flow interaction with topography, coastal dynamics, suppression of aircraft trailing vortices, convective and magnetohydrodynamic instability, and others. The results from this work have been reported in 11 refereed articles in prestigious journals, 10 articles in conference proceedings, and 37 formal presentations. The titles of these works are listed below, followed by an extended abstract of the work.

Publications in refereed journals:

Carnevale G.F., Briscolini M., Kloosterziel, R.C., and Vallis G.K., 1997, Three-dimensionally perturbed vortex tubes in a rotating flow, *J. Fluid Mech.* **341**, 127-163.

Carnevale G.F., Fuentes, O.V. and Orlandi, P., 1997, Inviscid dipole-vortex rebound from a wall or coast *J. Fluid Mech.* **351**, 75-103.

Orlandi, P. and Carnevale, G.F. 1999, Evolution of isolated vortices in a rotating fluid of finite depth, *J. Fluid Mech.* **381**, 239-269.

Kloosterziel, R.C. & Carnevale, G.F. 1999 On the evolution and saturation of instabilities of two-dimensional isolated circular vortices. *J. Fluid Mech.* **388**, 217-257.

Carnevale, G.F., Llewellyn Smith, S.G., Crisciani, F., Purini, R. and Serravall, R. 1999 Bifurcation of a coastal current at an escarpment. *J. Physical Oceanography* **29**, 969-985.

Carnevale, G.F., Briscolini, M. & Orlandi, P. 2001 Buoyancy to inertial range transition in forced stratified turbulence *J. Fluid Mech.* **427** 205-239.

Carnevale G.F., Cavallini, F. and Crisciani, F. 2001, Dynamic Boundary Conditions Revisited. *J. Physical Oceanography* **31**, 2489-2497.

Orlandi, P., Carnevale, G.F., Lele, S.K. & Shariff, K. 2001 Thermal Perturbation of Trailing Vortices European Journal of Mechanics B/Fluids *European Journal of Mechanics B/Fluids* **20** 511-524.

Carnevale, G.F., Orlandi, P., Zhou, Y. and Kloosterziel, R.C. 2002 Rotational suppression of Rayleigh-Taylor instability *J. Fluid Mech.* **457** 181-190.

Kloosterziel, R.C. and Carnevale, G.F. 2003 Closed-form linear stability conditions for rotating Rayleigh-Bénard convection with rigid, stress-free upper and lower boundaries *J. Fluid Mech.* **480** 25-42.

Kloosterziel, R.C. and Carnevale, G.F. 2003 Closed-form linear stability conditions for magneto-convection *J. Fluid Mech.* **490** 333-345.

Conference proceedings:

Orlandi, P., Carnevale, G.F., Lele, S.K., and Shariff, K. 1998, DNS study of the stability of trailing vortices, Proceedings of the 1998 Summer Program, Center for Turbulence Research, Stanford University pp. 187-208.

Carnevale G.F. and Briscolini, M., 1999, Large Eddy Simulation of Oceanic Fine Structure, (*'Aha Huliko'a, Internal Wave Modeling, Proceedings, Hawaiian Winter Workshop, University of Hawaii, January 19-22, 1999* eds. P. Muller and D. Henderson).

Orlandi, P. and Carnevale, G.F. 1999, Large eddy simulation of stratified flows. INI/ERCOFTAC Workshop on DIRECT AND LARGE-EDDY SIMULATION (Cambridge, May 1999)

Carnevale, G.F., Briscolini, M. & Orlandi, P. 2000 Transition from waves to turbulence in forced stratified flows (in *Fifth International Symposium on Stratified Flows*, University British Columbia, Vancouver, 10-13 July 2000, Ed. Lawrence, G.A., Pieters, R. and Yonemitsu, N.), (Pub. Dept. Civil Engineering, UBC. Vancouver. pp. 1272) Vol. 2, pp. 1179-1183.

Carnevale, G.F. and Orlandi, P., 2000 Propagation of Internal Wave Packets in the Thermocline, Proceedings of the 2000 Summer Program, Center for Turbulence Research, Stanford University pp. 119-130.

Carnevale G.F., Orlandi, P, Briscolini, M. & Kloosterziel, R.C., 2001, Simulations of internal-wave breaking and wave-packet propagation in the thermocline. (*'Aha Huliko'a, From stirring to mixing in a stratified ocean, Proceedings, Hawaiian Winter Workshop, University of Hawaii, January, 2001* eds. P. Muller and D. Henderson).

Dietrich, D., G.F. Carnevale, & P. Orlandi Adriatic simulations by DieCAST Proceedings of the 2002 Summer Program, Center for Turbulence Research, (ed. W.C. Reynolds and P. Moin), Stanford University.

Carnevale G.F., Orlandi, P, Briscolini, M. & Kloosterziel, R.C., 2002, 'Internal-wave-packet propagation and breaking' (in *Proceedings of the Nagoya Workshop 2001 on Statistical theories and computational approaches to turbulence*

Carnevale, G.F., Kloosterziel, R.C., Orlandi, P. and Zhou, Y. 2003 Rotational suppression of Rayleigh-Benard and Rayleigh-Taylor instability in Proceedings of the Symposium on Transport and Structural Formation combined with US/Japan JIFT workshop (Joint Interchange Fusion Theory) Structural Formation and Drift/MHD Turbulence Research Institute for Applied Mechanics Kyusho University Fukuoka, Japan

Carnevale, G.F., Kloosterziel, R.C., Orlandi, P., and Zhou, Y. 2003 'Effects of rotation on convective instability' in Nonlinear Processes in Geophysical Fluid Dynamics. O. U. Velasco Fuentes, J. Sheinbaum and J. Ochoa (editors). Kluwer Academic Publishers. Dordrecht, The Netherlands pp. 325-338.

Presentations:

Three-Dimensionally Perturbed Vortex Tubes in a Rotating Flow, USF, St. Petersburg, FL, 3/97

The stability of two-dimensional compound vortices, USF, St. Petersburg, FL, 3/97

The stability of two-dimensional compound vortices, University Southern California, 4/97

Vortex interactions with topography near a coast CICESE, Ensenada , Mexico, 5/97

Bifurcation of a coastal current at an escarpment, EPOC meeting, Stanford Sierra Camp, Fallen Leaf Lake, CA 9/97

Bifurcation of a coastal current at an escarpment, Istituto Talassografico di Trieste, Trieste, Italy 12/97

Instabilities of isolated vortices of finite depth, Theory group, SIO 1/98

Bifurcation of a coastal current at an escarpment, Physical Oceanography Research Div., University California San Diego 5/98

DNS study of the stability of trailing vortices, NASA/Center for Turbulence Research 7/98

Topographic effects on currents in the Adriatic Sea, ONR workshop on the Adriatic, Trieste 9/98

Laboratory Experiments on the Bifurcation of a coastal current at an escarpment, Istituto Talassografico di Trieste, Trieste, Italy 10/98

Mechanisms forcing vortices away from a coast, Istituto Talassografico di Trieste, Trieste, Italy 10/98

The stability of two-dimensional compound vortices, Istituto Talassografico di Trieste, Trieste, Italy 10/98

LES simulation of internal wave fine structure. Aha Huliko'a, Hawaiian Winter Workshop University of Hawaii, 1/99

From waves to turbulence in a stratified flow, Engineering Dept., University California San Diego 2/99

Transition from waves to turbulence in a stratified flow, Invited presentation at the Euromech Colloquium 396, Vortical Structures in Rotating and Stratified Fluids, Cortona, Italy, June, 6/99.

Transition from waves to turbulence in a stratified flow, Istituto Talassografico di Trieste, Trieste, Italy 1/00

Vortex Dynamics in Geophysical Flows Engineering Dept. University of Rome I 1/00

Coastal Flows Engineering Dept. University of Rome I 1/00

Relation between Weak Wave Interaction and Eddy Damped Markovian closure theories for systems with waves and turbulence LMFA/PEPIT-CPLG Workshop on Two-point closures and their applications Lyon, France 5/00

Buoyancy to inertial range transition in forced stratified turbulence AMS-IMS-SIAM Joint Summer Research Conference on Dispersive Wave Turbulence Mt. Holyoke 6/00

Transition from waves to turbulence in forced stratified flow, 5th International Symposium on Stratified Flows University British Columbia, Vancouver 7/00

Propagation and reflection of internal wave packets CTR Summer Research Program 2000 NASA/Center for Turbulence Research 7/00

Transition from waves to turbulence in forced stratified flow, University California San Diego PORD Theoretical Seminar 11/00

Internal-wave breaking in the thermocline University of Arizona Applied Math Dept. Tucson (inviter talk) 11/00

Overturning and mixing in the thermocline forced by wave-wave interactions Aha Huliko'a, Hawaiian Winter Workshop University of Hawaii, (inviter talk) 1/01

Overturning and mixing in the thermocline forced by wave-wave interactions Engineering Dept. Seminar University California San Diego 2/01

Overturning and mixing in the thermocline forced by wave-wave interactions PORD Seminar University California San Diego 2/01

Stability and Statistics of Flow over Topography, Chorin II, Workshop on Stochastic Climate Models, Chorin, Germany 7/2001 (invited talk)

Internal wave packets and the transition to turbulence International Workshop (10 - 13 October 2001) Statistical theories and computational approaches to turbulence : modern perspectives and application to global scale flows Nagoya, Japan 10/2001 (invited talk)

Internal wave packets and the transition to turbulence Oceanography Dept. National Taiwan University Taipei, Taiwan 10/2001 (invited seminar)

Transition from internal waves to turbulence National Center for Atmospheric Research Boulder, CO 10/2001 (Invited seminar)

Rotational suppression of Rayleigh-Taylor instability P.O. THEORETICAL SEMINAR University California San Diego 10/2001

Rotational suppression of Rayleigh-Taylor instability Physics Dept. University of Rome 'La Sapienza' 12/2001

Internal Waves, Transport Theories and Statistical Closures Mathematics of Subgrid-scale Phenomena in Atmospheric and Oceanic Flows IPAM (Institute for Pure and Applied Mathematics) UCLA 28/1/2002 - 5/2/2002 (Invited seminar)

Rotational suppression of Rayleigh-Benard and Rayleigh-Taylor instability Transport and Structural Formation and S/Japan JIFT workshop (Joint Interchange Fusion Theory) Structural Formation and Drift/MHD Turbulence Research Institute for Applied Mechanics Kyusho University Fukuoka, Japan 9/2002 (Invited talk)

Effects of rotation on convective instability Pedro Ripa Memorial Colloquium: Non-linear Processes in Geophysical Fluid Dynamics CICESE, Ensenada Mexico 10/2002 (Invited talk)

Extended Abstract:

Numerical experiments are used to study the evolution of perturbed vortex tubes in a rotating environment in order to better understand the process of two-dimensionalization of unsteady, rotating flows. We specifically consider non-axisymmetric perturbations to columnar vortices aligned along the axis of rotation. The basic unperturbed vortex is chosen to have a Gaussian cross-sectional vorticity distribution. The experiments cover a parameter space in which both the strength of the initial perturbation and the Rossby number are varied. The Rossby number is defined here as the ratio of the maximum amplitude of vorticity in the Gaussian vorticity profile to twice the ambient rotation rate. For small perturbations and small Rossby numbers, both cyclones and anticyclones behave similarly, relaxing rapidly back toward two-dimensional columnar vortices. For large perturbations and small Rossby numbers, a rapid instability occurs for both cyclones and anticyclones in which antiparallel vorticity is created. The tubes breakup and then reform again into columnar vortices parallel to the rotation axis (i.e. into a quasi-two-dimensional flow) through nonlinear processes. For Rossby numbers greater than one, even small perturbations result in the complete breakdown of the anticyclonic vortex through centrifugal instability, while cyclones remain stable. For a range of Rossby numbers greater than one, after the breakdown of the anticyclone, a new weaker anticyclone forms, with a small-scale

background vorticity of spectral shape given approximately by the -5/3 energy spectral law.

A vortex approaching a no-slip wall 'rebounds' due to the creation of vorticity at the wall in a viscous boundary layer. Here it is demonstrated that a purely inviscid mechanism can also produce vortex rebound from a slip wall. In inviscid vortex rebound, vortex tube stretching generates the necessary vorticity to allow rebound, eliminating the need for viscous vorticity generation. This vortex stretching mechanism is demonstrated through numerical simulations and laboratory experiments on dipole-vortex rebound from a boundary. In an application to oceanography, numerical simulations of both quasi-geostrophic and shallow water dynamics are used to show that the β-effect at an eastern boundary can produce this inviscid rebound. Through a series of numerical experiments in which the strength of the β-effect is varied, a formula for predicting the point of separation of the vortices from the boundary in a dipole-coast collision is deduced. Through simulations, the flux of vorticity and fluid away from the boundary is measured as a function of β and initial angle of incidence. It is found that, in contrast to viscous vortex rebound, which typically does not produce a flux of material away from the boundary farther than a distance comparable to the initial vortex radius, the β-induced rebound does carry fluid far from the coast. Laboratory experiments in a rotating tank are used to show that a sloping bottom can also provide an inviscid mechanism for dipole-vortex rebound from the wall of the tank under certain conditions. A relation determining the conditions under which inviscid or viscous processes will dominate in the rebound of the dipole from a boundary is obtained.

Laboratory experiments have shown that monopolar isolated vortices in a rotating flow undergo instabilities that result in the formation of multipolar vortex states such as dipoles and tripoles. In some cases the instability is entirely two-dimensional, with the vortices being vortex columns aligned along the direction of the ambient rotation at all times. In other cases, the vortex first passes through a highly turbulent three-dimensional state before eventually reorganizing into vortex columns. Through a series of three-dimensional numerical simulations, the roles that centrifugal instability, barotropic instability, and the bottom Ekman boundary layer play in these instabilities is investigated. Evidence is presented that the centrifugal instability can trigger the barotropic instabilities by the enhancement of vorticity gradients. It is shown that the bottom Ekman layer is not essential to these instabilities but can strongly modify the evolution.

Laboratory observations and numerical experiments have shown that a variety of compound vortices can emerge in two-dimensional flow due to the instability of isolated circular vortices. The simple geometrical features of these compound vortices suggest that their description may take a simple form if an appropriately chosen set of functions is used. We employ a set which is complete on the infinite plane for vorticity distributions with finite total enstrophy. Through projection of the vorticity equation (Galerkin method) and subsequent truncation we derive a dynamical system which is used to model the observed behavior in as simple as possible a fashion. It is found that at relatively low-order truncations the observed behavior is qualitatively captured by the dynamical system. We determine what the necessary ingredients are for saturation of instabilities at finite amplitude in terms of wave-wave interactions and feedback between various azimuthal components of the vorticity field.

The evolution of a coastal current as it encounters an escarpment depends strongly on

whether the geometry of the coast and escarpment is right or left 'handed,' independent of the direction of the coastal current. 'Handedness' is defined such that 'right-handed' means that when looking across the escarpment from the deep to the shallow side, the coast is found on the right. The essential aspects of the difference in behavior of the current in the two geometries are captured by a simple quasi-geostrophic model of coastal flow over a step. An exact analytic solution to the nonlinear stationary problem is obtained. Numerical simulations are used to examine the evolution from the initial encounter to the establishment of a stationary flow. The relevance of this research is discussed in light of recent results from laboratory experiments and oceanic observations.

The buoyancy range, which represents a transition from large-scale wave-dominated motions to small-scale turbulence in the oceans and the atmosphere, is investigated through large-eddy simulations. The model presented here uses a continual forcing based on large-scale standing internal-waves and has a spectral truncation in the isotropic inertial range. Evidence is presented for a break in the energy spectra from the anisotropic k^{-3} buoyancy range to the small-scale $k^{-5/3}$ isotropic inertial range. Density structures that form during wave breaking and periods of high strain rate are analyzed. Elongated vertical structures produced during periods of strong straining motion are found to collapse in the subsequent vertically-compressional phase of the strain resulting in a zone or patch of mixed fluid.

The applicability of the super-slip boundary condition in wind-driven quasi-geostrophic ocean circulation models is reexamined in the light of a new understanding of the effect of this boundary condition on energy conservation. A model is constructed with super-slip on the western boundary and free slip on the other boundaries. Both linear and nonlinear solutions are presented. Compared to the case with all free-slip boundaries, this new model gives a more energetic and narrower western boundary current, but otherwise the differences are not very great. A general criterion for energetically acceptable boundary conditions is also presented.

The possibility of diminishing the danger of vortices trailing behind aircraft through thermal forcing is investigated. It is shown that heating the vortices would have two beneficial effects. First, it would cause the vortices to descend more rapidly thus clearing the flight path more quickly. Second, it would cause the vortices to draw closer together, thus greatly increasing the growth rate of the short-wave instabilities that can ultimately destroy the vortices through cross-diffusion.

It is demonstrated that the growth of the mixing zone generated by Rayleigh-Taylor instability can be greatly retarded by the application of rotation, at least for low Atwood number flows for which the Boussinesq approximation is valid. This result is analyzed in terms of the effect of the Coriolis force on the vortex rings that propel the bubbles of fluid in the mixing zone.

The linear dynamics of rotating Rayleigh-Bénard convection with rigid, stress-free boundaries has been thoroughly investigated by Chandrasekhar (1961) who determined the marginal stability boundary and critical horizontal wavenumbers for the onset of convection and overstability as a function of the Taylor number T. No closed-form formulae appeared to exist and the results were tabulated numerically. However, by taking the Rayleigh number R as independent variable we have found remarkably simple expressions. When the Prandtl number $Pr \geq Pr_c = 0.67659$, the marginal stability boundary is described by the convection curve $T(R) = R\left[(R/R_c)^{1/2} - 1\right]$ where $R_c = (27/4)\pi^4$ is Rayleigh's famous critical value for the onset of steady convection in a non-rotating system ($T = 0$). For

$Pr < Pr_c$ the marginal stability boundary is determined by this curve until it intersects the overstability curve $T(R, Pr) = R\left[\left(\frac{1+Pr}{2^3 P^4}\right)^{1/2}(R/R_c)^{1/2} - \frac{1+Pr}{2Pr^2}\right]$. A simple expression for the intersection point is derived and also for the critical horizontal wavenumbers for which along the marginal stability boundary instability sets in either as steady convection or overstable oscillations. A simple formula is derived for the frequency of the oscillations. Further, we show that for high-enough rotation rates, with everything else the same, the higher the kinematic viscosity of the fluid, the more likely the system is to be unstable, and the higher thermal diffusivity, the more likely it is to be unstable. Finally, we show that if the fluid has zero viscosity the system is always unstable, in contradiction to Chandrasekhar's conclusion, and similarly if the thermal diffusivity is zero the system is always unstable.

Chandrasekhar (1961) extensively investigated the linear dynamics of Rayleigh-Bénard convection in an electrically conducting fluid exposed to a uniform vertical magnetic field and enclosed by rigid, stress-free, upper and lower boundaries. He determined the marginal stability boundary and critical horizontal wavenumbers for the onset of convection as a function of the Chandrasekhar number Q or Hartmann number squared. No closed-form formulae appeared to exist and the results were tabulated numerically. We have discovered simple expressions that concisely describe the stability properties of the system. When the Prandtl number Pr is greater than or equal to the magnetic Prandtl number Pm the marginal stability boundary is described by the curve $Q = \pi^{-2}\left[R - R_c^{1/3}R^{2/3}\right]$ where R is the Rayleigh number and $R_c = (27/4)\pi^4$ is Rayleigh's famous critical value for the onset of stationary convection in the absence of a magnetic field ($Q = 0$). When $Pm > Pr$ the marginal stability boundary is determined by this curve until intersected by the curve

$$Q = \frac{1}{\pi^2}\left[\frac{Pm^2(1+Pr)}{Pr^2(1+Pm)}R - \left(\frac{(1+Pr)(Pr+Pm)}{Pr^2}\right)^{1/3}\left(\frac{Pm^2(1+Pr)}{Pr^2(1+Pm)}\right)^{2/3} R_c^{1/3}R^{2/3}\right].$$

An expression for the intersection point is derived and also for the critical horizontal wavenumbers for which along the marginal stability boundary instability sets in either as stationary convection or in an oscillatory fashion. A simple formula is derived for the frequency of the oscillations. Also we show that in the limit of vanishing magnetic diffusivity, or infinite electrical conductivity, the system is unstable for sufficiently large R, contrary to Chandrasekhar's conclusion that the system must be stable for all R in this limit. Instability in this limit always sets in via overstability.